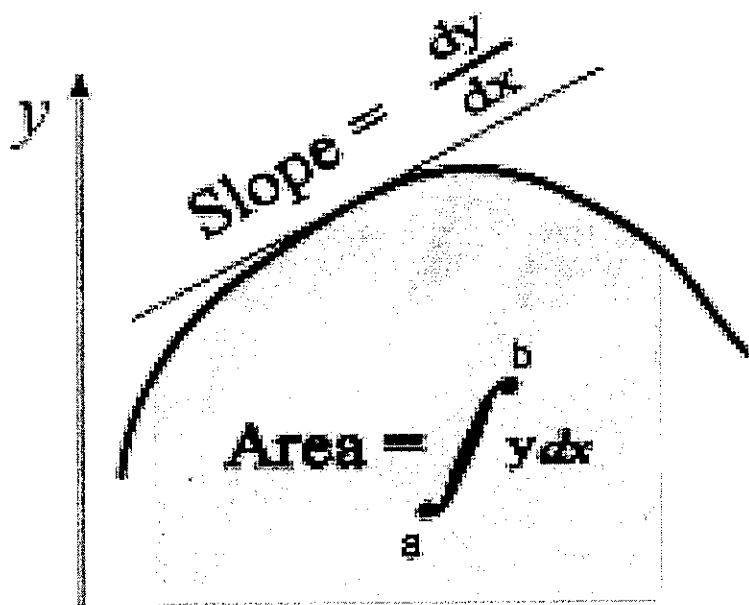


# *AP Calculus*

## *Summer Packet*



Dear AP Calculus Student,

Congratulations and welcome to AP Calculus! While the start of next year seems far away, it is important that you have the resources and materials necessary to begin Calculus available to you now so that you can make sure you are ready.

The attached packet contains a course introduction and list of previously studied topics that are essential for success in Calculus, along with reference sheets and practice problems (answer keys included) on concepts you are expected to know prior to starting this course. Please take some time to review the materials in this packet so that you are ready when school begins in the fall.

AP Calculus AB and BC, like other AP courses, are designed to help you expand your knowledge and deepen your understanding of concepts at a high level. Therefore, students are expected to be dedicated to the course, committed to putting in the time necessary to be successful, and responsible for keeping up with current material while maintaining previously discussed concepts.

Enjoy your summer and I look forward to a working with you next year.

Best regards,  
Mrs. Gutheil

## AP Calculus Introduction

Calculus is a branch of advanced mathematics that deals with problems that cannot be solved with ordinary algebra. This includes rate problems where slopes are not fixed and area and volume problems involving irregular objects. The AP Calculus courses are designed to introduce students to two main branches of calculus (differential and integral calculus) and related topics. An essential component of these courses is the continued development of critical thinking skills and problem solving skills. Therefore, students will be expected to show appropriate work and to explain results using correct calculus justifications.

Below is a list of prerequisite skills (from Algebra I, Geometry, Algebra II and PreCalculus) which are important to your success in AP Calculus. Topics with an asterisk (\*) are required skills for AP Calculus BC only. In order to be prepared to meet the challenges of AP Calculus, it is recommended that students review the topics below before the start of the new school year. Students will be expected to incorporate their knowledge on these topics into calculus questions, many *without the use of a calculator.* Without these prerequisite skills, you will struggle to correctly solve problems next year even though you understand the calculus concepts. If you find that you are in need of a refresher on one or more of the topics below, there are some suggested online resources at the end of the list.

Following the list of prerequisite topics are several handouts with information you are expected to know and some practice problems to work on. Solutions to the practice problems are included so that you can check your work.

### Prerequisite Topics:

1. Fundamentals of Algebra
  - a. Rules of exponents
  - b. Factoring (including GCF, difference of two squares, trinomial, grouping, sum/difference of perfect cubes)
  - c. Polynomial long and synthetic division
  - d. Operations with fractions
  - e. Rationalizing, simplifying and expanding expressions
  - f. Simplifying complex fractions
  - g. Solving equations, systems of equations and inequalities (linear and quadratic)

## 2. Functions

- a. Domain and range
- b. Composition of functions
- c. Operations with functions
- d. Parent functions
- e. Average rate of change
- f. Point-slope form of a line
- g. Inverse functions
- h. Odd/Even functions
- i. Piecewise functions
- j. Finding solutions with and without a calculator (including quadratic, radical, rational, absolute value, and polynomial equations)

## 3. Exponentials and Logarithms

- a. Rules and properties
- b. Solving equations
- c. Common and natural logarithms

## 4. Rational Functions

- a. Asymptotes
- b. Intercepts
- c. Holes
- d. End behavior

## 5. Trigonometry

- a. Right triangle trig
- b. Exact values of special angles and the Unit Circle
- c. Reciprocal trig functions
- d. Inverse trig functions
- e. Solving equations
- f. Pythagorean identities

## 6. Limits

- a. Techniques for evaluation (substitution, simplification, rationalization)
- b. One-sided limits
- c. Determining limits from a graph
- d. Limits as  $x \rightarrow \pm\infty$
- e. Limits that approach infinity
- f. Limits that do not exist (DNE)

7. Derivatives
  - a. Constant rule
  - b. Power rule
  - c. Product rule
  - d. Quotient rule
  - e. Chain rule
  
8. Area and Volume Formulas
  - a. Area for circle, square, rectangle, triangle, equilateral triangle
  - b. Volume for a cone, sphere, cylinder, rectangular prism, cube
  
9. Partial Fractions\*
  
10. Sequences and Series\*
  - a. Identify arithmetic and geometric sequences and series
  - b. Sigma notation for a series
  - c. Finite and infinite series
  - d. Sum of a series
  
11. Polar Equations\*
  - a. Convert between polar and rectangular coordinates/equations
  - b. Graph polar equations

Online Resources:

[www.khanacademy.com](http://www.khanacademy.com)  
[www.purplemath.com](http://www.purplemath.com)  
[www.brightstorm.com](http://www.brightstorm.com)  
[www.coolmath.com](http://www.coolmath.com)

# Things to Know for Calculus

## TRIGONOMETRY

### Trig Functions

$$\sin \theta = \frac{\text{opp}}{\text{hyp}}$$

$$\cos \theta = \frac{\text{adj}}{\text{hyp}}$$

$$\tan \theta = \frac{\text{opp}}{\text{adj}}$$

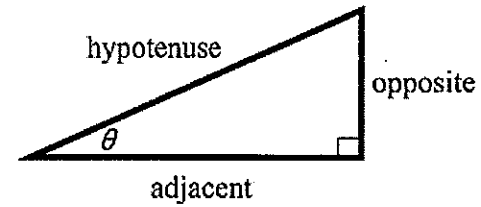
### Reciprocal Functions

$$\csc \theta = \frac{1}{\sin \theta} = \frac{\text{hyp}}{\text{opp}}$$

$$\sec \theta = \frac{1}{\cos \theta} = \frac{\text{hyp}}{\text{adj}}$$

$$\cot \theta = \frac{1}{\tan \theta} = \frac{\text{adj}}{\text{opp}}$$

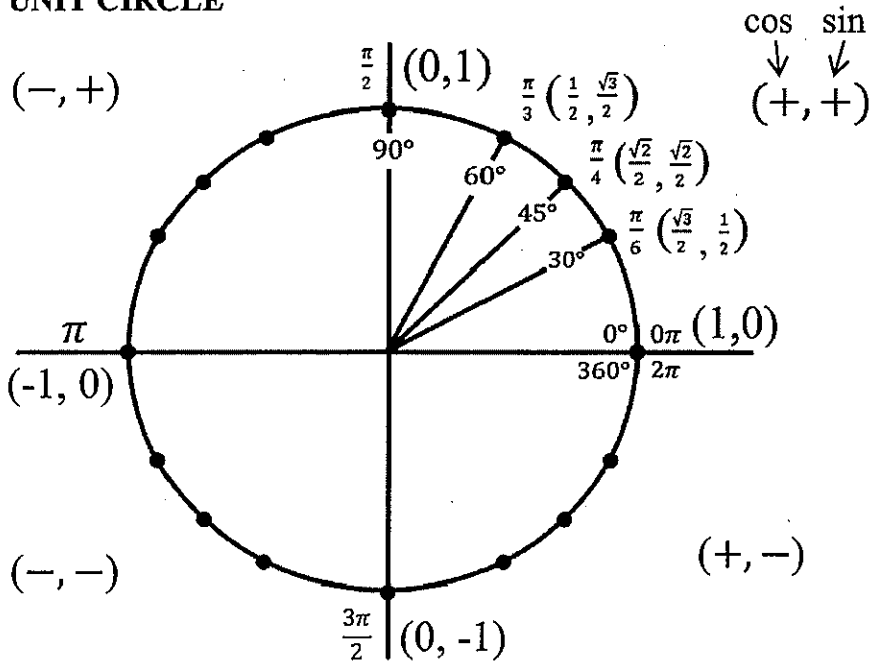
SOH-CAH-TOA



### TEST ONLY USES RADIANS!

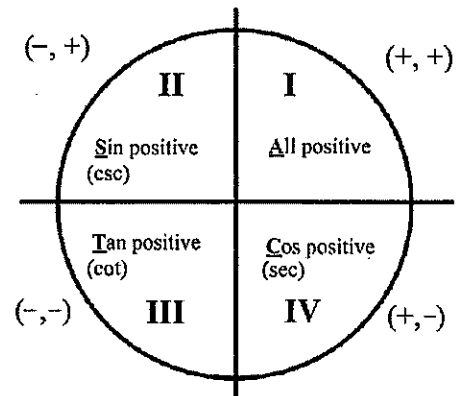
Must know trig values of special angles  $0\pi, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$  using Unit Circle or Special Right Triangles.

### UNIT CIRCLE



To help remember the signs in each quadrant

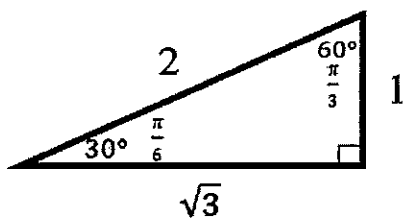
All Students Take Calculus



### SPECIAL RIGHT TRIANGLES

$30^\circ - 60^\circ - 90^\circ$  Triangles

Which are  $\frac{\pi}{6} - \frac{\pi}{3} - \frac{\pi}{2}$  Triangles

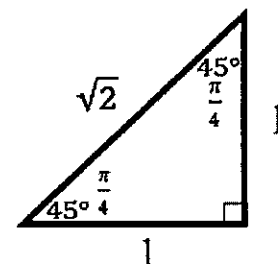


Find  $\tan\left(\frac{\pi}{6}\right)$

$$\tan\left(\frac{\pi}{6}\right) = \frac{\text{opp}}{\text{adj}} = \frac{1}{\sqrt{3}} \text{ simplify to } \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$45^\circ - 45^\circ - 90^\circ$  Triangles

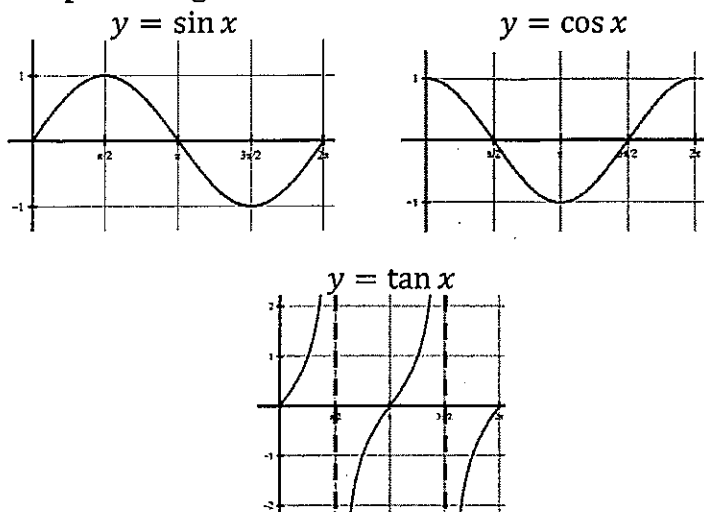
Which are  $\frac{\pi}{4} - \frac{\pi}{4} - \frac{\pi}{2}$  Triangles



Find  $\sin\left(\frac{\pi}{4}\right)$

$$\sin\left(\frac{\pi}{4}\right) = \frac{\text{opp}}{\text{hyp}} = \frac{1}{\sqrt{2}} \text{ simplify to } \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

## Graphs of trig functions



## Inverse Trig Function

$\sin^{-1}\theta$  is the same as  $\arcsin \theta$

$\sin^{-1}\theta = \left(\frac{\sqrt{3}}{2}\right)$  means what angle has a sine value of  $\frac{\sqrt{3}}{2}$

that means  $\theta = \frac{\pi}{3} \pm 2\pi n$  or  $\frac{2\pi}{3} \pm 2\pi n$

Since  $\theta$  has infinite answers then it isn't a function. Bummer. To make it a function we define inverses like:

sin/csc and tan/cot use quadrant I and IV for inverses  
cos/sec use quadrant I and II for inverses

So...  $\theta = \frac{\pi}{3}$  because it is in the first quadrant

## Trig Identities

There are a bunch, but you really only need to know Pythagorean Identity.  $\sin^2 x + \cos^2 x = 1$

Subtract  $\sin^2 x$  to get  $\cos^2 x = 1 - \sin^2 x$  or subtract  $\cos^2 x$  to get  $\sin^2 x = 1 - \cos^2 x$

Divide by  $\sin^2 x$  to get  $1 + \cot^2 x = \csc^2 x$  or divide by  $\cos^2 x$  to get  $\tan^2 x + 1 = \sec^2 x$

## GEOMETRY

### FORMULAS

#### AREA

$$\text{Triangle} = \frac{1}{2}bh$$

$$\text{Circle} = \pi r^2$$

$$\text{Trapezoid} = \frac{1}{2}(b_1 + b_2)h$$

#### CIRCUMFERENCE

$$\text{Circle} = 2\pi r$$

#### SURFACE AREA

$$\text{Sphere} = 4\pi r^2$$

#### LATERAL AREA

$$\text{Cylinder} = 2\pi rh$$

#### VOLUME

$$\text{Sphere} = \frac{4}{3}\pi r^3$$

$$\text{Cylinder} = \pi r^2 h$$

$$\text{Cone} = \frac{1}{3}\pi r^2 h$$

$$\text{Prism} = Bh$$

$$\text{Pyramid} = \frac{1}{3}Bh$$

$B$  is the area of the base

#### DISTANCE FORMULA

The distance between two points  $(x_1, y_1)$  and  $(x_2, y_2)$  is  $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

## ALGEBRA

### Linear Functions

Slope

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

y-intercept Form

(slope-intercept Form)

$$y = mx + b$$

Point Slope Form

$$y - y_1 = m(x - x_1)$$

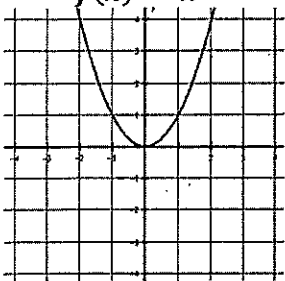
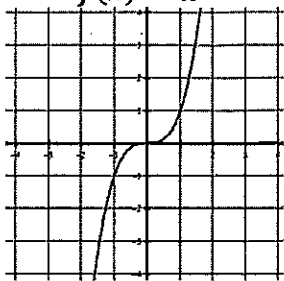
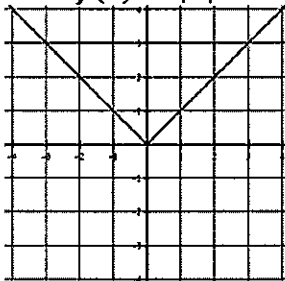
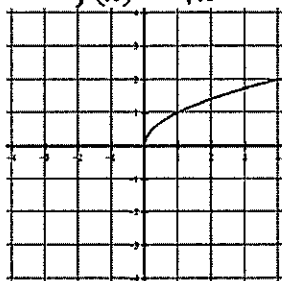
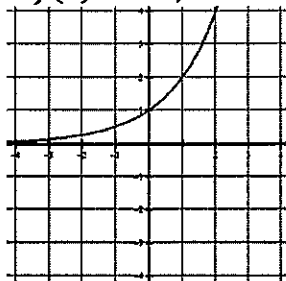
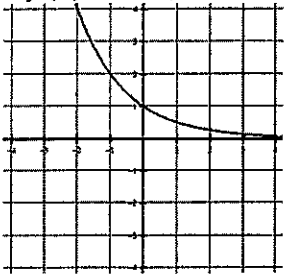
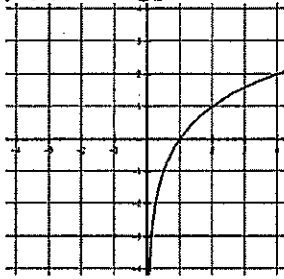
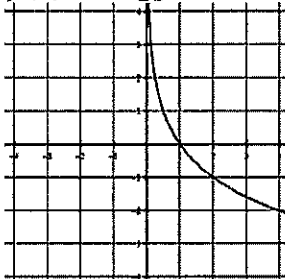
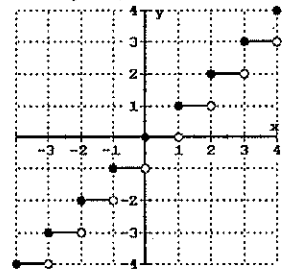
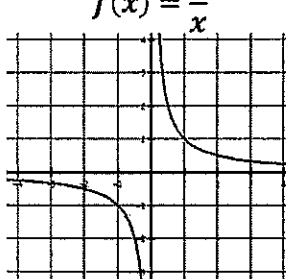
Parallel Lines

Have the same slope

Perpendicular Lines

Have the opposite reciprocal slopes

## Functions

<p>Quadratic Function</p> $f(x) = x^2$  $y = a(x - h)^2 + k$	<p>Cubic Function</p> $f(x) = x^3$  $y = a(x - h)^3 + k$	<p>Absolute Value</p> $f(x) =  x $  $y = a x - h  + k$	<p>Square Root Function</p> $f(x) = \sqrt{x}$  $y = a\sqrt{x - h} + k$	<p>Exponential Function</p> $f(x) = b^x, b > 1$  $y = a \cdot b^{(x-h)} + k$
<p>Exponential Function</p> $f(x) = b^x, b < 1$  $y = a \cdot b^{(x-h)} + k$	<p>Logarithmic Function</p> $f(x) = \log_b x, b > 1$  $y = a \log_b(x - h) + k$	<p>Logarithmic Function</p> $f(x) = \log_b x, b < 1$  $y = a \log_b(x - h) + k$	<p>Greatest Integer</p> $f(x) = \lceil x \rceil$  $y = a \lceil x - h \rceil + k$	<p>Rational Function</p> $f(x) = \frac{1}{x}$  $y = \frac{a}{x - h} + k$

## Translations

All functions move the same way!

Given the parent function  $y = x^2$

Move up 4  
 $y = x^2 + 4$

Move down 3  
 $y = x^2 - 3$

Move left 2  
 $y = (x + 2)^2$

Move right 1  
 $y = (x - 1)^2$

Move left 2 and down 3  
 $y = (x + 2)^2 - 3$

To flip (reflect) the function vertically  $y = -x^2$   
 To flip (reflect) the function horizontally  $y = (-x)^2$

So  $f(x) = -\sqrt{x - 3} + 1$  is a square root function reflected vertically, shifted right 3 and up 1

## Notation

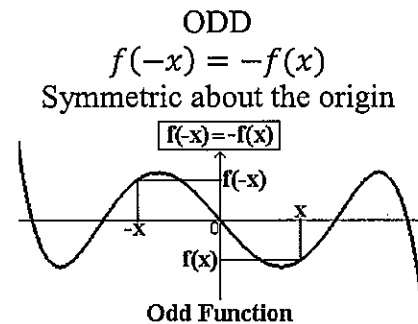
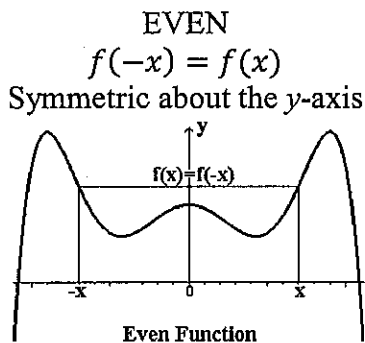
Notice open parenthesis ( ) versus closed [ ]

Inequality	Interval
$-3 < x \leq 5$	$(-3, 5]$
$-3 \leq x \leq 5$	$[-3, 5]$
$-3 < x < 5$	$(-3, 5)$
$-3 \leq x < 5$	$[-3, 5)$

Infinity is always open parenthesis

Inequality	Interval
$x < 3$	$(-\infty, 3)$
$x \leq 3$ or $x > 5$	$(-\infty, 3] \cup (5, \infty)$
$x \neq 3$	$(-\infty, 3) \cup (3, \infty)$
all Real numbers	$(-\infty, \infty)$

## Even and Odd Functions





## Domain and Range

Domain = all possible  $x$  values

Range = all possible  $y$  values

Algebraically

You can't divide by zero  
You can't square root a negative

Graphically

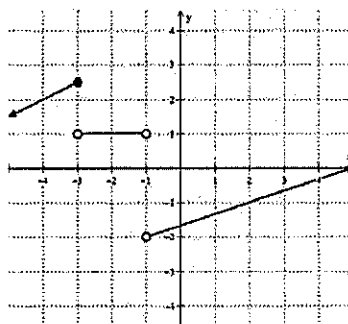
Just look at it

$$y = \sqrt{2x + 5}$$

$$D: [-\frac{5}{2}, \infty)$$

$$y = \frac{x^2 - 1}{x^2 + 7x + 12}$$

$$D: (-\infty, -4)(-4, -3)(-3, \infty)$$



$$D: (-\infty, -1)(-1, 5]$$

$$R: (-\infty, 2.5]$$

## Finding zeros

Must be able to factor and use the quadratic formula:  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

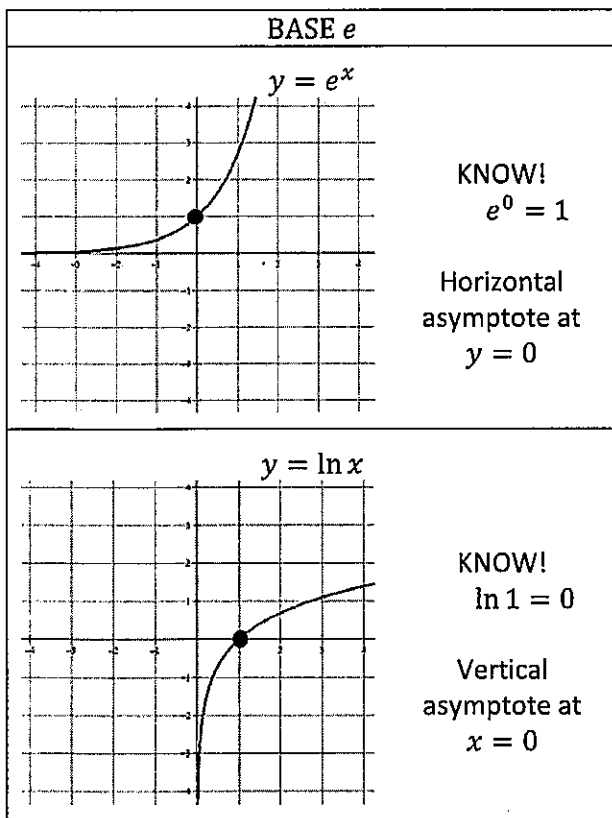
## Special products

Sum of cubes:  $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

Difference of cubes:  $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$

## Exponential and Logarithmic Properties

The exponential function  $b^x$  of base  $b$  is one-to-one which means it has an inverse which is called the logarithmic function of base  $b$  or logarithm of base  $b$  which is denoted  $\log_b x$  which reads "the logarithm of base  $b$  of  $x$ " or "log base  $b$  of  $x$ ". So...



$$y = \log_b x \longleftrightarrow x = b^y$$

Exponential		Logarithmic
$b^x b^y = b^{x+y}$	Product Rule	$\log_b xy = \log_b x + \log_b y$
$\frac{b^x}{b^y} = b^{x-y}$	Quotient Rule	$\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$
$(b^x)^y = b^{xy}$	Power Rule	$\log_b x^y = y \log_b x$
$b^{-x} = \frac{1}{b^x}$		$\log_b \left(\frac{1}{x}\right) = -\log_b x$
$b^0 = 1$		$\log_b 1 = 0$
$b^1 = b$		$\log_b b = 1$
	Change of Base	$\log_b x = \frac{\log_c x}{\log_c b}$
	Natural Log	$\log_e x = \ln x$
	Common Log	$\log_{10} x = \log x$

# Calculus - SUMMER PACKET

NAME: \_\_\_\_\_

Summer + Math = (Best Summer Ever)<sup>2</sup>

## NO CALCULATOR!!!

Given  $f(x) = x^2 - 2x + 5$ , find the following.

1.  $f(-2) =$

2.  $f(x + 2) =$

3.  $f(x + h) =$

Use the graph  $f(x)$  to answer the following.

4.  $f(0) =$

$f(4) =$

$f(-1) =$

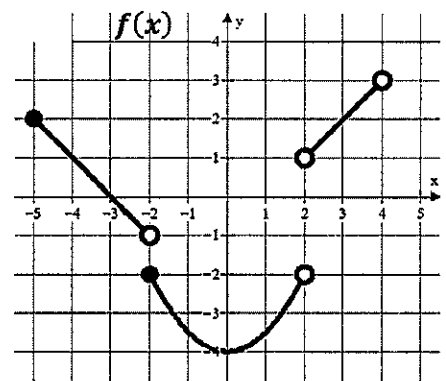
$f(-2) =$

$f(2) =$

$f(3) =$

$f(x) = 2$  when  $x = ?$

$f(x) = -3$  when  $x = ?$



Write the equation of the line meets the following conditions. Use point-slope form.

$y - y_1 = m(x - x_1)$

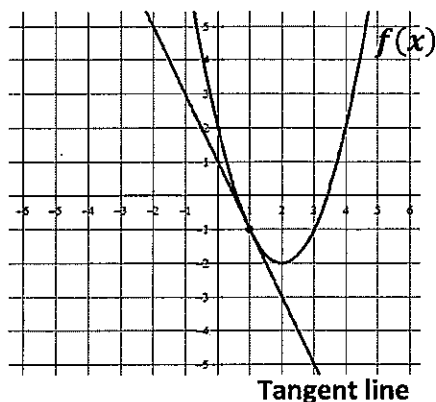
5. slope = 3 and  $(4, -2)$

6.  $m = -\frac{3}{2}$  and  $f(-5) = 7$

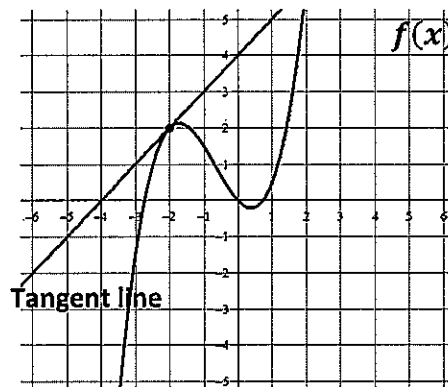
7.  $f(4) = -8$  and  $f(-3) = 12$

Write the equation of the tangent line in point slope form.  $y - y_1 = m(x - x_1)$

8. The line tangent to  $f(x)$  at  $x = 1$



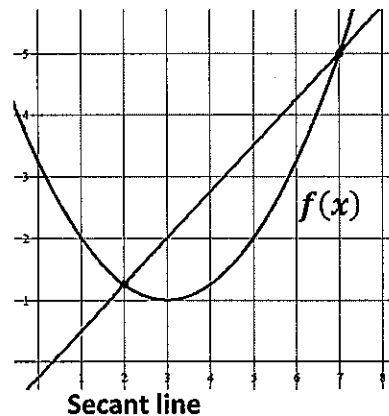
9. The line tangent to  $f(x)$  at  $x = -2$



**MULTIPLE CHOICE!** Remember slope =  $\frac{y_2 - y_1}{x_2 - x_1}$

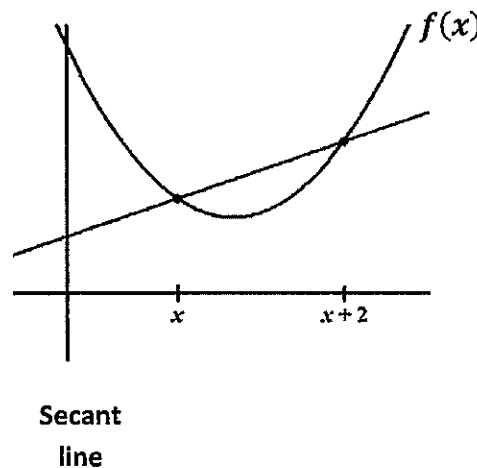
10. Which choice represents the slope of the secant line shown?

- A)  $\frac{7-2}{f(7)-f(2)}$     B)  $\frac{f(7)-2}{7-f(2)}$     C)  $\frac{7-f(2)}{f(7)-2}$     D)  $\frac{f(7)-f(2)}{7-2}$



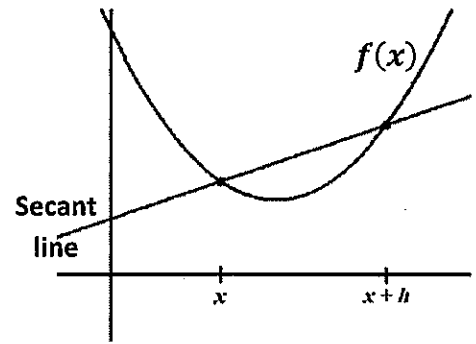
11. Which choice represents the slope of the secant line shown?

- A)  $\frac{f(x)-f(x+2)}{x+2-x}$     B)  $\frac{f(x+2)-f(x)}{x+2-x}$     C)  $\frac{f(x+2)-f(x)}{x-(x+2)}$
- D)  $\frac{x+2-x}{f(x)-f(x+2)}$



12. Which choice represents the slope of the secant line shown?

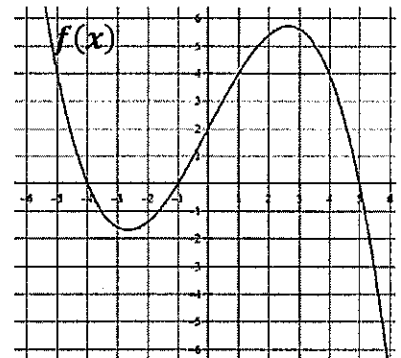
- A)  $\frac{f(x+h)-f(x)}{x-(x+h)}$       B)  $\frac{x-(x+h)}{f(x+h)-f(x)}$       C)  $\frac{f(x+h)-f(x)}{x+h-x}$
- D)  $\frac{f(x)-f(x+h)}{x+h-x}$



13. Which of the following statements about the function  $f(x)$  is true?

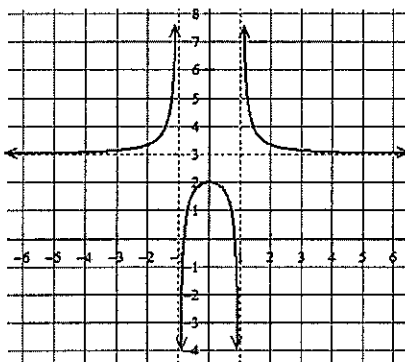
- I.  $f(2) = 0$   
 II.  $(x + 4)$  is a factor of  $f(x)$   
 III.  $f(5) = f(-1)$

- (A) I only  
 (B) II only  
 (C) III only  
 (D) I and III only  
 (E) II and III only



**Find the domain and range (express in interval notation). Find all horizontal and vertical asymptotes.**

14.



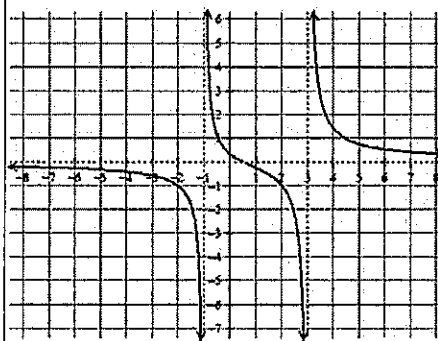
Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

15.



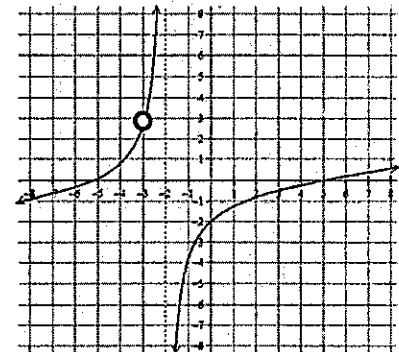
Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

16.



Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

**MULTIPLE CHOICE!**

17. Which of the following functions has a vertical asymptote at  $x = 4$  ?

(A)  $\frac{x+5}{x^2-4}$

(B)  $\frac{x^2-16}{x-4}$

(C)  $\frac{4x}{x+1}$

(D)  $\frac{x+6}{x^2-7x+12}$

(E) None of the above

18. Consider the function:  $f(x) = \frac{x^2-5x+6}{x^2-4}$ . Which of the following statements is true?

I.  $f(x)$  has a vertical asymptote of  $x = 2$ II.  $f(x)$  has a vertical asymptote of  $x = -2$ III.  $f(x)$  has a horizontal asymptote of  $y = 1$ 

(A) I only

(B) II only

(C) I and III only

(D) II and III only

(E) I, II and III

**Rewrite the following using rational exponents. Example:  $\frac{1}{\sqrt[3]{x^2}} = x^{-\frac{2}{3}}$**

19.  $\sqrt[5]{x^3} + \sqrt[5]{2x}$

20.  $\sqrt{x+1}$

21.  $\frac{1}{\sqrt{x+1}}$

22.  $\frac{1}{\sqrt{x}} - \frac{2}{x}$

23.  $\frac{1}{4x^3} + \frac{1}{2}\sqrt[4]{x^3}$

24.  $\frac{1}{4\sqrt{x}} - 2\sqrt{x+1}$

**Write each expression in radical form and positive exponents. Example:  $x^{-\frac{2}{3}} + x^{-2} = \frac{1}{\sqrt[3]{x^2}} + \frac{1}{x^2}$**

25.  $x^{-\frac{1}{2}} - x^{\frac{3}{2}}$

26.  $\frac{1}{2}x^{-\frac{1}{2}} + x^{-1}$

27.  $3x^{-\frac{1}{2}}$

28.  $(x+4)^{-\frac{1}{2}}$

29.  $x^{-2} + x^{\frac{1}{2}}$

30.  $2x^{-2} + \frac{3}{2}x^{-1}$

**Need to know basic trig functions in RADIANS! We never use degrees. You can either use the Unit Circle or Special Triangles to find the following.**

31. $\sin \frac{\pi}{6}$	32. $\cos \frac{\pi}{4}$	33. $\sin 2\pi$
34. $\tan \pi$	35. $\sec \frac{\pi}{2}$	36. $\cos \frac{\pi}{6}$
37. $\sin \frac{\pi}{3}$	38. $\sin \frac{3\pi}{2}$	39. $\tan \frac{\pi}{4}$
40. $\csc \frac{\pi}{2}$	41. $\sin \pi$	42. $\cos \frac{\pi}{3}$
43. Find $x$ where $0 \leq x \leq 2\pi$ , $\sin x = \frac{1}{2}$	44. Find $x$ where $0 \leq x \leq 2\pi$ , $\tan x = 0$	45. Find $x$ where $0 \leq x \leq 2\pi$ , $\cos x = -1$

**Solve the following equations. Remember  $e^0 = 1$  and  $\ln 1 = 0$ .**

46. $e^x + 1 = 2$	47. $3e^x + 5 = 8$	48. $e^{2x} = 1$
49. $\ln x = 0$	50. $3 - \ln x = 3$	51. $\ln(3x) = 0$
52. $x^2 - 3x = 0$	53. $e^x + xe^x = 0$	54. $e^{2x} - e^x = 0$

Solve the following trig equations where  $0 \leq x \leq 2\pi$ .

55.  $\sin x = \frac{1}{2}$

56.  $\cos x = -1$

57.  $\cos x = \frac{\sqrt{3}}{2}$

58.  $2\sin x = -1$

59.  $\cos x = \frac{\sqrt{2}}{2}$

60.  $\cos\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2}$

61.  $\tan x = 0$

62.  $\sin(2x) = 1$

63.  $\sin\left(\frac{x}{4}\right) = \frac{\sqrt{3}}{2}$

For each function, determine its domain and range.

Function	Domain	Range
64. $y = \sqrt{x - 4}$		
65. $y = (x - 3)^2$		
66. $y = \ln x$		
67. $y = e^x$		
68. $y = \sqrt{4 - x^2}$		

Simplify.

69.  $\frac{\sqrt{x}}{x}$

70.  $e^{\ln x}$

71.  $e^{1+\ln x}$

72. $\ln 1$	73. $\ln e^7$	74. $\log_3 \frac{1}{3}$
75. $\log_{1/2} 8$	76. $\ln \frac{1}{2}$	77. $27^{\frac{2}{3}}$
78. $(5a^{2/3})(4a^{3/2})$	79. $\frac{4xy^{-2}}{12x^{\frac{1}{3}}y^{-5}}$	80. $(4a^{5/3})^{3/2}$

If  $f(x) = \{(3, 5), (2, 4), (1, 7)\}$ ,  $g(x) = \sqrt{x - 3}$ ,  
 $h(x) = \{(3, 2), (4, 3), (1, 6)\}$ ,  $k(x) = x^2 + 5$ , then determine each of the following.

81. $(f + h)(1)$	82. $(k - g)(5)$	83. $f(h(3))$
84. $g(k(7))$	85. $h(3)$	86. $g(g(9))$
87. $f^{-1}(4)$	88. $k^{-1}(x)$	
89. $k(g(x))$	90. $g(f(2))$	



Summer + Math = (Best Summer Ever)<sup>2</sup>

## NO CALCULATOR!!!

Given  $f(x) = x^2 - 2x + 5$ , find the following.

1.  $f(-2) =$

$$f(-2) = (-2)^2 - 2(-2) + 5$$

$$f(-2) = 4 + 4 + 5$$

$$f(-2) = 13$$

2.  $f(x+2) =$

$$x^2 + 2x + 5$$

3.  $f(x+h) =$

$$(x+h)^2 - 2(x+h) + 5$$

$$(x+h)(x+h) - 2x - 2h + 5$$

$$x^2 + xh + xh + h^2 - 2x - 2h + 5$$

$$x^2 + 2xh + h^2 - 2x - 2h + 5$$

Use the graph  $f(x)$  to answer the following.

4.  $f(0) = -4$

$f(4) =$  DNE (Does not exist) or undefined

$f(-1) = -3.5$

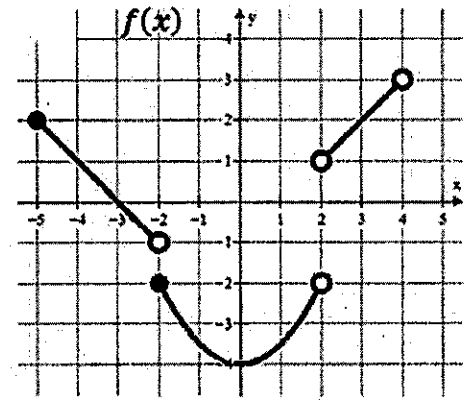
$f(-2) = -2$

$f(2) =$  DNE (Does not exist) or undefined

$f(3) = 2$

$f(x) = 2$  when  $x = ?$   
-5 and 3

$f(x) = -3$  when  $x = ?$   
-1.5 and 1.5



Write the equation of the line meets the following conditions. Use point-slope form.

$$y - y_1 = m(x - x_1)$$

5. slope = 3 and  $(4, -2)$

$$y - (-2) = 3(x - 4)$$

$$y + 2 = 3(x - 4)$$

6.  $m = -\frac{3}{2}$  and  $f(-5) = 7$

$$y - 7 = -\frac{3}{2}(x + 5)$$

7.  $f(4) = -8$  and  $f(-3) = 12$

$$m = \frac{12 - (-8)}{-3 - 4} = \frac{20}{-7}$$

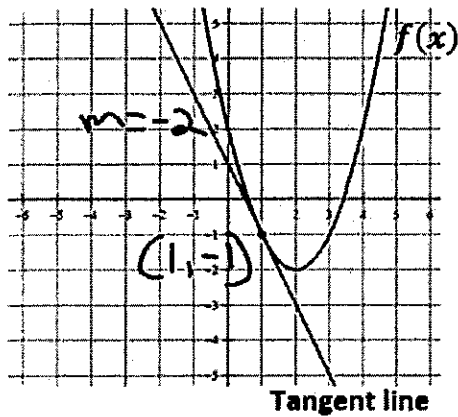
$$y - 12 = -\frac{20}{7}(x + 3)$$

or

$$y + 8 = -\frac{20}{7}(x - 4)$$

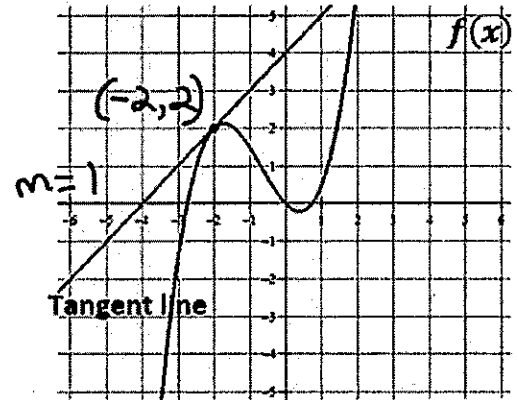
Write the equation of the tangent line in point slope form.  $y - y_1 = m(x - x_1)$

8. The line tangent to  $f(x)$  at  $x = 1$



$$y + 1 = -2(x - 1)$$

9. The line tangent to  $f(x)$  at  $x = -2$



$$y - 2 = 1(x + 2)$$

**MULTIPLE CHOICE!** Remember slope =  $\frac{y_2 - y_1}{x_2 - x_1}$

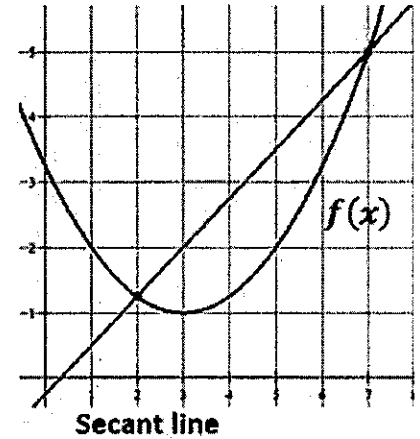
10. Which choice represents the slope of the secant line shown?

A)  $\frac{7-2}{f(7)-f(2)}$

B)  $\frac{f(7)-2}{7-f(2)}$

C)  $\frac{7-f(2)}{f(7)-2}$

D)  $\frac{f(7)-f(2)}{7-2}$



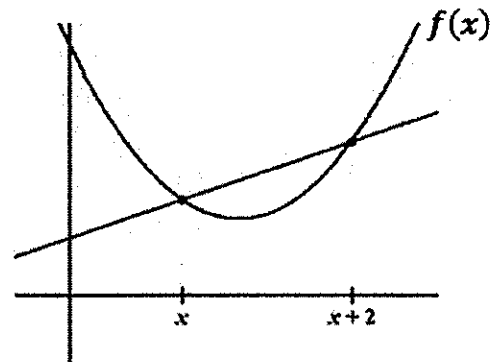
11. Which choice represents the slope of the secant line shown?

A)  $\frac{f(x)-f(x+2)}{x+2-x}$

B)  $\frac{f(x+2)-f(x)}{x+2-x}$

C)  $\frac{f(x+2)-f(x)}{x-(x+2)}$

D)  $\frac{x+2-x}{f(x)-f(x+2)}$



Secant line

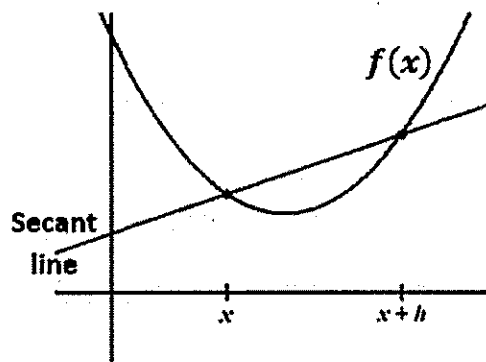
12. Which choice represents the slope of the secant line shown?

A)  $\frac{f(x+h)-f(x)}{x-(x+h)}$

B)  $\frac{x-(x+h)}{f(x+h)-f(x)}$

C)  $\frac{f(x+h)-f(x)}{x+h-x}$

D)  $\frac{f(x)-f(x+h)}{x+h-x}$



13. Which of the following statements about the function  $f(x)$  is true?

I.  $f(2) = 0$

II.  $(x + 4)$  is a factor of  $f(x)$

III.  $f(5) = f(-1)$

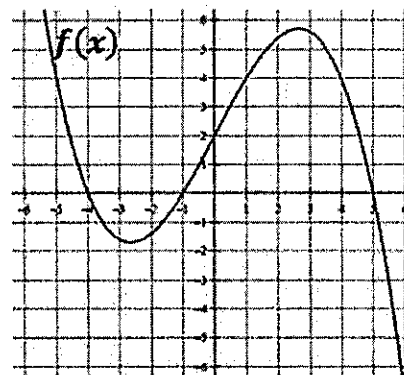
(A) I only

(B) II only

(C) III only

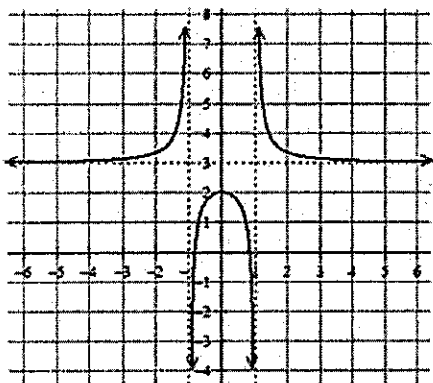
(D) I and III only

(E) II and III only



**Find the domain and range (express in interval notation). Find all horizontal and vertical asymptotes.**

14.



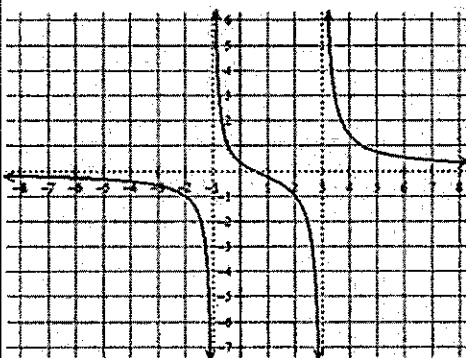
Domain:  $(-\infty, -1) \cup (-1, 1) \cup (1, \infty)$

Range:  $(-\infty, 2] \cup (3, \infty)$

Horizontal Asymptote(s):  $y = 3$

Vertical Asymptotes(s):  $x = 1$   
 $x = -1$

15.



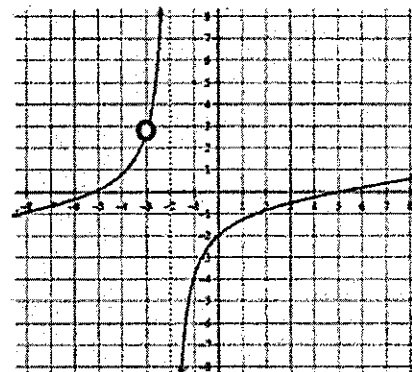
Domain:  $(-\infty, -1) \cup (-1, 3) \cup (3, \infty)$

Range:  $(-\infty, \infty)$

Horizontal Asymptote(s):  $y = 0$

Vertical Asymptotes(s):  $x = -1$   
 $x = 3$

16.



Domain:  $(-\infty, -3) \cup (-3, -2) \cup (-2, \infty)$

Range:  $(-\infty, \infty)$

Horizontal Asymptote(s): none

Vertical Asymptotes(s):  $x = -2$

**MULTIPLE CHOICE!**

17. Which of the following functions has a vertical asymptote at  $x = 4$  ?

(A)  $\frac{x+5}{x^2-4}$

(B)  $\frac{x^2-16}{x-4}$

(C)  $\frac{4x}{x+1}$

(D)  $\frac{x+6}{x^2-7x+12}$

(E) None of the above

18. Consider the function:  $f(x) = \frac{x^2-5x+6}{x^2-4}$ . Which of the following statements is true?

- I.  $f(x)$  has a vertical asymptote of  $x = 2$
- II.  $f(x)$  has a vertical asymptote of  $x = -2$
- III.  $f(x)$  has a horizontal asymptote of  $y = 1$

(A) I only

(B) II only

(C) I and III only

(D) II and III only

(E) I, II and III

**Rewrite the following using rational exponents. Example:  $\frac{1}{\sqrt[3]{x^2}} = x^{-\frac{2}{3}}$**

19.  $\sqrt[5]{x^3} + \sqrt[5]{2x}$   
 $x^{\frac{3}{5}} + (2x)^{\frac{1}{5}}$

20.  $\sqrt{x+1}$   
 $(x+1)^{\frac{1}{2}}$

21.  $\frac{1}{\sqrt{x+1}}$   
 $(x+1)^{-\frac{1}{2}}$

22.  $\frac{1}{\sqrt{x}} - \frac{2}{x}$   
 $x^{-\frac{1}{2}} - 2x^{-1}$

23.  $\frac{1}{4x^3} + \frac{1}{2}\sqrt[4]{x^3}$   
 $\frac{1}{4}x^{-3} + \frac{1}{2}x^{\frac{3}{4}}$

24.  $\frac{1}{4\sqrt{x}} - 2\sqrt{x+1}$   
 $\frac{1}{4}x^{-\frac{1}{2}} - 2(x+1)^{\frac{1}{2}}$

**Write each expression in radical form and positive exponents. Example:  $x^{\frac{2}{3}} + x^{-2} = \sqrt[3]{x^2} + \frac{1}{x^2}$**

25.  $x^{\frac{1}{2}} - x^{\frac{3}{2}}$   
 $\sqrt{x} - \sqrt{x^3}$

26.  $\frac{1}{2}x^{-\frac{1}{2}} + x^{-1}$   
 $\frac{1}{2\sqrt{x}} + \frac{1}{x}$

27.  $3x^{\frac{1}{2}}$   
 $3\sqrt{x}$

28.  $(x+4)^{-\frac{1}{2}}$   
 $\frac{1}{\sqrt{x+4}}$

29.  $x^{-2} + x^{\frac{1}{2}}$   
 $\frac{1}{x^2} + \sqrt{x}$

30.  $2x^{-2} + \frac{3}{2}x^{-1}$   
 $\frac{2}{x^2} + \frac{3}{2x}$

**Need to know basic trig functions in RADIANS! We never use degrees. You can either use the Unit Circle or Special Triangles to find the following.**

31.  $\sin \frac{\pi}{6}$

$\frac{1}{2}$

32.  $\cos \frac{\pi}{4}$

$\frac{\sqrt{2}}{2}$

33.  $\sin 2\pi$

0

34.  $\tan \pi$

0

35.  $\sec \frac{\pi}{2}$

undefined

36.  $\cos \frac{\pi}{6}$

$\frac{\sqrt{3}}{2}$

37.  $\sin \frac{\pi}{3}$

$\frac{\sqrt{3}}{2}$

38.  $\sin \frac{3\pi}{2}$

-1

39.  $\tan \frac{\pi}{4}$

1

40.  $\csc \frac{\pi}{2}$

1

41.  $\sin \pi$

0

42.  $\cos \frac{\pi}{3}$

$\frac{1}{2}$

43. Find  $x$  where  $0 \leq x \leq 2\pi$ ,

$\sin x = \frac{1}{2}$

$\frac{\pi}{6}$  and  $\frac{5\pi}{6}$

44. Find  $x$  where  $0 \leq x \leq 2\pi$ ,

$\tan x = 0$

0 and  $\pi$

45. Find  $x$  where  $0 \leq x \leq 2\pi$ ,

$\cos x = -1$

$\pi$

**Solve the following equations. Remember  $e^0 = 1$  and  $\ln 1 = 0$ .**

46.  $e^x + 1 = 2$

$e^x = 1$

$\ln(e^x) = \ln(1)$

$x = 0$

47.  $3e^x + 5 = 8$

$3e^x = 3$

$e^x = 1$

$\ln e^x = \ln 1$

$x = 0$

48.  $e^{2x} = 1$

$\ln e^{2x} = \ln(1)$

$2x = 0$

$x = 0$

49.  $\ln x = 0$

$e^e$

$x = 1$

50.  $3 - \ln x = 3$

$-\ln x = 0$

$\ln x = 0$

$e^e$

$x = 1$

51.  $\ln(3x) = 0$

$e^e$

$3x = 1$

$x = \frac{1}{3}$

52.  $x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0$   $x = 3$

53.  $e^x + xe^x = 0$

$e^x(1+x) = 0$

$e^x = 0$   $1+x = 0$

not possible

$x = -1$

54.  $e^{2x} - e^x = 0$

$e^x(e^x - 1) = 0$

$e^x = 0$

not possible

$e^x - 1 = 0$

$e^x = 1$

$x = 0$

Solve the following trig equations where  $0 \leq x \leq 2\pi$ .

55.  $\sin x = \frac{1}{2}$

$x = \frac{\pi}{6}$  and  $\frac{5\pi}{6}$

56.  $\cos x = -1$

$x = \pi$

57.  $\cos x = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{6}$  and  $\frac{11\pi}{6}$

58.  $2\sin x = -1$

$\sin x = -\frac{1}{2}$

$x = \frac{7\pi}{6}$  and  $\frac{11\pi}{6}$

59.  $\cos x = \frac{\sqrt{2}}{2}$

$x = \frac{\pi}{4}$  and  $\frac{7\pi}{4}$

60.  $\cos\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2}$

$\frac{x}{2} = \frac{\pi}{6}$        $\frac{x}{2} = \frac{11\pi}{6}$

$x = \frac{\pi}{3}$  and  $\frac{11\pi}{3}$

61.  $\tan x = 0$

$\frac{\sin x}{\cos x} = 0 \rightarrow \sin x = 0$

$x = 0$  and  $x = \pi$

62.  $\sin(2x) = 1$

$2x = \frac{\pi}{2}$

$x = \frac{\pi}{4}$

63.  $\sin\left(\frac{x}{4}\right) =$

$\frac{x}{4} = \frac{\pi}{3}$

$x = \frac{4\pi}{3}$

For each function, determine its domain and range.

Function	Domain	Range
64. $y = \sqrt{x-4}$	$x \geq 4$	$y \geq 0$
65. $y = (x-3)^2$	$\mathbb{R}$ all real numbers	$y \geq 0$
66. $y = \ln x$	$x > 0$	$\mathbb{R}$
67. $y = e^x$	$\mathbb{R}$	$y > 0$
68. $y = \sqrt{4-x^2}$	$-2 \leq x \leq 2$	$0 \leq y \leq 2$

Simplify.

69.  $\frac{\sqrt{x}}{x}$

$\frac{x^{\frac{1}{2}}}{x^1}$   
 $x^{\frac{1}{2}-1}$   
 $x^{-\frac{1}{2}}$

$\frac{1}{\sqrt{x}}$

70.  $e^{\ln x}$

$x$

71.  $e^{1+\ln x}$

$e^1 \cdot e^{\ln x}$

$ex$

72.  $\ln 1$

$$0$$

73.  $\ln e^7$

$$7$$

74.  $\log_3 \frac{1}{3}$

$$\log_3 3^{-1}$$
  
$$-1$$

75.  $\log_{1/2} 8$   
 $\log_{1/2} (\frac{1}{2})^{-3}$

$$-3$$

76.  $\ln \frac{1}{2}$  Calculator needed

$$\approx -0.693$$

77.  $27^{2/3}$

$$\sqrt[3]{27^2}$$
  
$$3^2$$
  
$$9$$

78.  $(5a^{2/3})(4a^{3/2})$

$$20a^{\frac{2}{3} + \frac{3}{2}}$$

$$20a^{\frac{13}{6}}$$

79.  $\frac{4xy^{-2}}{12x^{-1/3}y^{-5}}$   
 $\frac{1}{3} x^{1 - (-1/3)} y^{-2 - (-5)}$

$$\frac{1}{3} x^{\frac{4}{3}} y^3$$

80.  $(4a^{5/3})^{3/2}$

$$\sqrt{4^3} a^{\frac{5}{3} \cdot \frac{3}{2}}$$

$$8a^{\frac{5}{2}}$$

If  $f(x) = \{(3, 5), (2, 4), (1, 7)\}$      $g(x) = \sqrt{x-3}$     then determine each of the following.  
 $h(x) = \{(3, 2), (4, 3), (1, 6)\}$      $k(x) = x^2 + 5$

81.  $(f+h)(1)$

$$f(1) + h(1)$$
  
$$7 + 6$$

$$13$$

82.  $(k-g)(5)$

$$k(5) - g(5)$$
  
$$(25+5) - (\sqrt{2})$$

$$30 - \sqrt{2}$$

83.  $f(h(3))$

$$f(2)$$

$$4$$

84.  $g(k(7))$

$$g(7^2+5)$$

$$g(54)$$

$$\sqrt{54-3} = \sqrt{51}$$

85.  $h(3)$

$$2$$

86.  $g(g(9))$

$$g(\sqrt{6})$$

$$\sqrt{\sqrt{6}-3}$$

87.  $f^{-1}(4)$

$$2$$

88.  $k^{-1}(x)$

$$x = y^2 + 5$$
  
$$x - 5 = y^2$$

$$y = \sqrt{x-5}$$

89.  $k(g(x)) = (\sqrt{x-3})^2 + 5$

$$x-3+5$$

$$x+2$$

90.  $g(f(2))$

$$g(4) = \sqrt{4-3}$$

$$1$$

# 1 Review – Limits

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points.

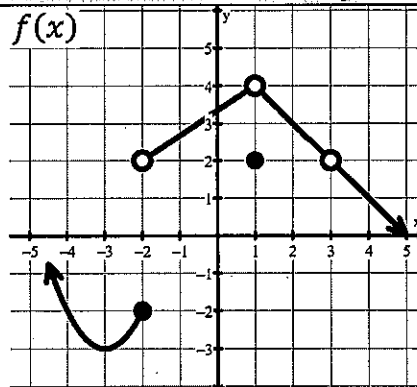
## 1.1 Limits Graphically:

What is a limit?

The **y-value** a function approaches at a given **x-value**.

**Give the value of each statement. If the value does not exist, write "does not exist" or "undefined."**

- |                                    |                                       |
|------------------------------------|---------------------------------------|
| 1. $\lim_{x \rightarrow 3} f(x) =$ | 5. $\lim_{x \rightarrow 2} f(x) =$    |
| 2. $\lim_{x \rightarrow 1} f(x) =$ | 6. $\lim_{x \rightarrow -2^+} f(x) =$ |
| 3. $f(3) =$                        | 7. $f(1) =$                           |
| 4. $f(-2) =$                       | 8. $\lim_{x \rightarrow -2^-} f(x) =$ |



## 1.2 Limits Analytically:

**Finding a limit:**

1. Direct Substitution.
2. Simplify and then try direct substitution.
  - a. Factor and Cancel.
  - b. Rationalize if you see square roots.

**Special Trig Limits:**

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = \quad \text{or} \quad \lim_{x \rightarrow 0} \frac{x}{\sin x} =$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = \quad \text{or} \quad \lim_{x \rightarrow 0} \frac{\cos x - 1}{x} =$$

We will cover these during the first week of school.

**Evaluate each limit.**

9.  $\lim_{x \rightarrow -4} (2x^2 + 3x - 2)$

10.  $\lim_{x \rightarrow 1} \sqrt{7x + 42}$

11.  $\lim_{x \rightarrow 13} 2$

12.  $\lim_{x \rightarrow 10} \frac{x^2 - 5x - 50}{x - 10}$



$$13. \lim_{x \rightarrow 0} \frac{\sqrt{x+19} - \sqrt{19}}{x}$$

$$14. \lim_{x \rightarrow 0} \frac{\frac{1}{x+1} - 1}{x}$$

$$15. \lim_{x \rightarrow 0} \frac{\sin(7x)}{11x}$$

$$16. \lim_{x \rightarrow 0} \frac{\sin^2(3x)}{\sin^2(5x)}$$

### 1.3 Asymptotes:

#### Vertical Asymptotes:

If the denominator equals 0, then there is a hole or a vertical asymptote. If the factor does not cancel, then it's a vertical asymptote.

One-sided limits at vertical asymptotes approach  $-\infty$  or  $\infty$ .

#### Horizontal asymptotes:

$\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)}$  will produce a horizontal asymptote at

- $y = 0$  if  $g$  increases faster than  $f$ .
- $y = \frac{a}{b}$  if  $g$  and  $f$  are increasing at the relative same amount where  $a$  and  $b$  are the coefficients of the fastest growing terms.

Don't forget to check the left and right sides when looking for horizontal asymptotes.

Evaluate each limit.			Find all horizontal asymptotes.
17. $\lim_{x \rightarrow \infty} \frac{4x^5 - 2x^2 + 3}{3x^2 + 2x^5 - x^4}$	18. $\lim_{x \rightarrow \infty} x^5 3^{-x}$	19. $\lim_{x \rightarrow \infty} \sin \frac{x + 3\pi x^2}{2x^2}$	20. $f(x) = \frac{\sqrt{16x^6 + x^3} + 5x}{5x^3 - 8x}$

### 1.4 Continuity:

#### Types of Discontinuities:

1. Removable (hole).
2. Discontinuity due to vertical asymptote.
3. Jump discontinuity.

#### Finding Domain:

Restrictions occur with two scenarios:

1. Denominators can't be zero.
2. Even radicals can't be negative.

Name: Solutions

Date: \_\_\_\_\_

Period: \_\_\_\_\_

**Review****1 Review – Limits**

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points.

**1.1 Limits Graphically:**

What is a limit?

The **y-value** a function approaches at a given **x-value**.**Give the value of each statement. If the value does not exist, write "does not exist" or "undefined."**

1.  $\lim_{x \rightarrow 3} f(x) = 2$

5.  $\lim_{x \rightarrow 2} f(x) = 3$

2.  $\lim_{x \rightarrow 1} f(x) = 4$

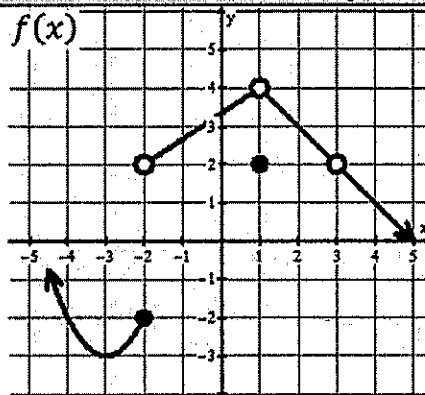
6.  $\lim_{x \rightarrow -2^+} f(x) = 2$

3.  $f(3) = \text{DNE}$

7.  $f(1) = 2$

4.  $f(-2) = -2$

8.  $\lim_{x \rightarrow -2^-} f(x) = -2$

**1.2 Limits Analytically:****Finding a limit:**

1. Direct Substitution.
2. Simplify and then try direct substitution.
  - a. Factor and Cancel.
  - b. Rationalize if you see square roots.
3. L'Hôpital's Rule (for indeterminate forms  $\frac{0}{0}$  or  $\frac{\infty}{\infty}$ )

**Special Trig Limits:**

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \quad \text{or} \quad \lim_{x \rightarrow 0} \frac{x}{\sin x} = 1$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0 \quad \text{or} \quad \lim_{x \rightarrow 0} \frac{\cos x - 1}{x} = 0$$

**Evaluate each limit.**

9.  $\lim_{x \rightarrow -4} (2x^2 + 3x - 2)$

$$2(-4)^2 + 3(-4) - 2$$

$$32 - 12 - 2$$

**18**

10.  $\lim_{x \rightarrow 1} \sqrt{7x + 42}$

$$\sqrt{7(1) + 42}$$

$$\sqrt{49}$$

$$7$$

11.  $\lim_{x \rightarrow 13} 2$

**2**

12.  $\lim_{x \rightarrow 10} \frac{x^2 - 5x - 50}{x - 10}$

$$\frac{(x-10)(x+5)}{x-10}$$

$$10 + 5 = 15$$

$$13. \lim_{x \rightarrow 0} \frac{\sqrt{x+19} - \sqrt{19}}{x} \cdot \frac{\sqrt{x+19} + \sqrt{19}}{\sqrt{x+19} + \sqrt{19}} = \lim_{x \rightarrow 0} \frac{(x+19) - (19)}{x(\sqrt{x+19} + \sqrt{19})}$$

$$\lim_{x \rightarrow 0} \frac{x}{x(\sqrt{x+19} + \sqrt{19})} = \frac{1}{\sqrt{0+19} + \sqrt{19}} = \frac{1}{2\sqrt{19}} = \frac{\sqrt{19}}{38}$$

$$14. \lim_{x \rightarrow 0} \frac{\frac{1}{x+1} - 1}{x} \cdot \frac{x+1}{x+1} = \lim_{x \rightarrow 0} \frac{1 - (x+1)}{x(x+1)}$$

$$\lim_{x \rightarrow 0} \frac{-x}{x(x+1)} = \lim_{x \rightarrow 0} \frac{-1}{0+1} = -1$$

$$15. \lim_{x \rightarrow 0} \frac{\sin(7x)}{11x} = \lim_{x \rightarrow 0} \frac{1}{11} \cdot \frac{\sin(7x)}{x} \cdot \frac{7}{7}$$

$$\lim_{x \rightarrow 0} \frac{7}{11} \cdot \frac{\sin(7x)}{7x} = \frac{7}{11}$$

$$16. \lim_{x \rightarrow 0} \frac{\sin^2(3x)}{\sin^2(5x)}$$

$$\lim_{x \rightarrow 0} \frac{\sin(3x)}{3x} \cdot \frac{\sin(3x)}{3x} \cdot \frac{5x}{\sin(5x)} \cdot \frac{5x}{\sin(5x)} \cdot \frac{3}{5} \cdot \frac{3}{5} = \frac{9}{25}$$

### 1.3 Asymptotes:

#### Vertical Asymptotes:

If the denominator equals 0, then there is a hole or a vertical asymptote. If the factor does not cancel, then it's a vertical asymptote.

One-sided limits at vertical asymptotes approach  $-\infty$  or  $\infty$ .

#### Horizontal asymptotes:

$\lim_{x \rightarrow \infty} \frac{f(x)}{g(x)}$  will produce a horizontal asymptote at

- $y = 0$  if  $g$  increases faster than  $f$ .
- $y = \frac{a}{b}$  if  $g$  and  $f$  are increasing at the relative same amount where  $a$  and  $b$  are the coefficients of the fastest growing terms.

Don't forget to check the left and right sides when looking for horizontal asymptotes.

Evaluate each limit.			Find all horizontal asymptotes.
17. $\lim_{x \rightarrow \infty} \frac{4x^5 - 2x^2 + 3}{3x^2 + 2x^5 - x^4}$	18. $\lim_{x \rightarrow \infty} x^5 3^{-x}$	19. $\lim_{x \rightarrow \infty} \sin \frac{x + 3\pi x^2}{2x^2}$	20. $f(x) = \frac{\sqrt{16x^4 + x^2 + 5x}}{5x^2 - 8x}$
$\frac{4}{2} = 2$	$\lim_{x \rightarrow \infty} \frac{x^5}{3^x}$ (BIG!) 0	$\sin\left(\frac{3\pi}{2}\right)$ -1	as $x \rightarrow \infty$ , $y = \frac{4}{5}$ as $x \rightarrow -\infty$ , $y = -\frac{4}{5}$

### 1.4 Continuity:

#### Types of Discontinuities:

1. Removable (hole).
2. Discontinuity due to vertical asymptote.
3. Jump discontinuity.

#### Finding Domain:

Restrictions occur with two scenarios:

1. Denominators can't be zero.
2. Even radicals can't be negative.

Name: \_\_\_\_\_ Date: \_\_\_\_\_ Period: \_\_\_\_\_

## 2 Review – The Derivative

Reviews do NOT cover all material from the lessons but will hopefully remind you of key points.

### 2.1 Average Rate of Change

A continuous function  $f(x)$  on the interval  $[a, b]$  has an average rate of change of

$$\frac{f(b) - f(a)}{b - a} \quad \text{or} \quad \frac{f(a) - f(b)}{a - b}$$

This is also the **SLOPE** of the **TANGENT** line.

**Find the average rate of change for each function on the given interval. Use units when necessary.**

1.  $w(t) = 5t^2 - 5t + 1; [-2, 1]$

2.  $s(x) = \frac{x+5}{3}; [1, 7]$

3.  $B(t) = \cos\left(\frac{\pi}{3}t\right); \left[\frac{3}{2}, 6\right]$

$B$  represents wild boar  
 $t$  represents weeks

### 2.2 Definition of the Derivative

#### Definition of the derivative:

This limit gives an expression that calculates the *instantaneous* rate of change (slope of the tangent line) of  $f(x)$  at any given  $x$ -value.

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

#### Derivative at a point:

Finding the derivative at a specific  $x$ -value. We will call this value  $c$ .

$$f'(c) = \lim_{h \rightarrow 0} \frac{f(c+h) - f(c)}{h}$$

or

$$f'(c) = \lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c}$$

Find the derivative using limits. WRITE SMALL!!!

4.  $y = 2x^2 + 3x - 1$

Create an equation of the tangent line of  $f$  at the given point. Leave in point-slope.

5.  $f(x) = -2x^3 + 3x$ ;  
 $f'(x) = -6x^2 + 3$ ;  $x = -1$

Identify the original function  $f(x)$ , and what value of  $c$  to evaluate  $f'(c)$

6.  $\lim_{h \rightarrow 0} \frac{-(3+h)^2 + (3+h) - 4 + (10)}{h}$

$f(x) =$  \_\_\_\_\_

$c =$  \_\_\_\_\_

7.  $\lim_{x \rightarrow 5} \frac{(4x - 2x^3) + (230)}{x - 5}$

$f(x) =$  \_\_\_\_\_

$c =$  \_\_\_\_\_

## 2 Review – The Derivative

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### 2.1 Average Rate of Change

A continuous function  $f(x)$  on the interval  $[a, b]$  has an average rate of change of

$$\frac{f(b) - f(a)}{b - a} \quad \text{or} \quad \frac{f(a) - f(b)}{a - b}$$

This is also the SLOPE of the TANGENT line.

**Find the average rate of change for each function on the given interval. Use units when necessary.**

1.  $w(t) = 5t^2 - 5t + 1; [-2, 1]$

$w(-2) = 31$   
 $w(1) = 1$

$$\frac{31 - 1}{-2 - 1} = \frac{30}{-3} = -10$$

-10

2.  $s(x) = \frac{x+5}{3}; [1, 7]$

$s(1) = 2$   
 $s(7) = 4$

$$\frac{2 - 4}{1 - 7} = \frac{-2}{-6} = \frac{1}{3}$$

$\frac{1}{3}$

3.  $B(t) = \cos\left(\frac{\pi}{3}t\right); \left[\frac{3}{2}, 6\right]$   
*B* represents wild boar  
*t* represents weeks

$B\left(\frac{3}{2}\right) = \cos\left(\frac{\pi}{2}\right) = 0$   
 $B(6) = \cos(2\pi) = 1$

$$\frac{0 - 1}{\frac{3}{2} - 6} = \frac{-1}{-\frac{9}{2}} = \frac{2}{9}$$

$\frac{2}{9}$  boar per week

### 2.2 Definition of the Derivative

**Definition of the derivative:**

This limit gives an expression that calculates the *instantaneous* rate of change (slope of the tangent line) of  $f(x)$  at any given  $x$ -value.

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

**Derivative at a point:**

Finding the derivative at a specific  $x$ -value. We will call this value  $c$ .

$$f'(c) = \lim_{h \rightarrow 0} \frac{f(c+h) - f(c)}{h}$$

or

$$f'(c) = \lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c}$$

Find the derivative using limits. WRITE SMALL!!

4.  $y = 2x^2 + 3x - 1$

$$y' = \lim_{h \rightarrow 0} \frac{2(x+h)^2 + 3(x+h) - 1 - [2x^2 + 3x - 1]}{h}$$

$$\frac{\cancel{2x^2} + 4xh + h^2 + \cancel{3x} + 3h - 1 - \cancel{2x^2} - \cancel{3x} + 1}{h}$$

$$y' = \lim_{h \rightarrow 0} \frac{h(4x + h + 3)}{h}$$

$$y' = 4x + 3$$

Create an equation of the tangent line of  $f$  at the given point. Leave in point-slope.

5.  $f(x) = -2x^3 + 3x$ ;

$f'(x) = -6x^2 + 3$ ;  $x = -1$

$$f(-1) = -1$$

$$f'(-1) = -3$$

$$y + 1 = -3(x + 1)$$

Identify the original function  $f(x)$ , and what value of  $c$  to evaluate  $f'(c)$ .

6.  $\lim_{h \rightarrow 0} \frac{-(3+h)^2 + (3+h) - 4 + (10)}{h}$

$f(x) = -x^2 + x - 4$

$c = 3$

7.  $\lim_{x \rightarrow 5} \frac{(4x - 2x^3) + (130)}{x - 5}$

$f(x) = 4x - 2x^3$

$c = 5$

## Constant and Power Rule Notes

**Constant & Power Rules****Constant Rule:**

The derivative of  $f(x) = c$  (where  $c$  is a constant) is  $f'(x) = 0$ .

**Power Rule:**

The derivative of  $f(x) = x^n$  is  $f'(x) = nx^{n-1}$  (It may be necessary to rewrite some functions in order to apply the power rule more easily, use the algebra you know to help.)

1.  $y = x^9$

5.  $f(x) = \frac{1}{x^4}$

2.  $y = -3$

6.  $y = \sqrt[4]{x^3}$

3.  $y = 2x^{12}$

7.  $y = \sqrt{x}$

4.  $f(x) = \pi$

8.  $f(x) = \frac{4}{x^2}$



## Constant and Power Rule Notes

**Constant & Power Rules****Constant Rule:**

The derivative of  $f(x) = c$  (where  $c$  is a constant) is  $f'(x) = 0$ .

**Power Rule:**

The derivative of  $f(x) = x^n$  is  $f'(x) = nx^{n-1}$  (It may be necessary to rewrite some functions in order to apply the power rule more easily, use the algebra you know to help.)

$$1. \quad y = x^9$$

$$\frac{dy}{dx} = 9x^8$$

$$2. \quad y = -3$$

$$\frac{dy}{dx} = 0$$

$$3. \quad y = 2x^{12}$$

$$\frac{dy}{dx} = 24x^{11}$$

$$4. \quad f(x) = \pi$$

$$f'(x) = 0$$

$$5. \quad f(x) = \frac{1}{x^4} \xrightarrow{\text{rewrite}} f(x) = x^{-4}$$

$$f'(x) = -4x^{-3} \text{ or } \frac{-4}{x^3}$$

$$6. \quad y = \sqrt[4]{x^3} \xrightarrow{\text{rewrite}} y = x^{3/4}$$

$$\frac{dy}{dx} = \frac{3}{4}x^{-1/4} \text{ or } \frac{3}{4\sqrt[4]{x}}$$

$$7. \quad y = \sqrt{x} \xrightarrow{\text{rewrite}} y = x^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2}x^{-1/2} \text{ or } \frac{1}{2\sqrt{x}}$$

$$8. \quad f(x) = \frac{4}{x^2} \xrightarrow{\text{rewrite}} f(x) = 4x^{-2}$$

$$f'(x) = -8x^{-3}$$

## Product & Quotient Rules

### Product Rule:

The derivative of  $f(x)g(x)$  is  $f'(x)g(x) + f(x)g'(x)$

1.  $f(x) = (2x^3)(5x+1)$

2.  $f(x) = 2x^{1/2}(3x-1)$

3.  $f(x) = (\frac{2}{3}x^3 + 6x - 2)(4\sqrt{x} - 6)$

**Quotient Rule:**

The derivative of  $\frac{f(x)}{g(x)}$  is  $\frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

1.  $f(x) = \frac{x^2 + 3}{x^2 - 4}$

2.  $f(x) = \frac{x+1}{x^2 + 2x + 2}$

3.  $f(x) = \left(\frac{x+4}{x+3}\right)(2x+5)$

4.  $f(x) = \frac{x^2 + c^2}{x^2 - c^2}$

\*  $c$  is a constant

### Product & Quotient Rules

#### Product Rule:

The derivative of  $f(x)g(x)$  is  $f'(x)g(x) + f(x)g'(x)$

1.  $f(x) = (2x^3)(5x+1)$

$$f'(x) = (2x^3)(5) + (5x+1)(6x^2)$$

$$f'(x) = 10x^3 + 30x^3 + 6x^2$$

$$f'(x) = 40x^3 + 6x^2$$

2.  $f(x) = 2x^{1/2}(3x-1)$

$$f'(x) = 2x^{1/2}(3) - (3x-1)(x^{-1/2})$$

$$f'(x) = 6x^{1/2} - 3x^{1/2} + x^{-1/2}$$

$$f'(x) = 3x^{1/2} + \frac{1}{x^{1/2}} \quad \text{or} \quad 3\sqrt{x} + \frac{1}{\sqrt{x}}$$

3.  $f(x) = \left(\frac{2}{3}x^3 + 6x - 2\right)(4\sqrt{x} - 6)$   $\left(4x^{1/2} - 6\right)$

$$f'(x) = \left(\frac{2}{3}x^3 + 6x - 2\right)(2x^{-1/2}) + (4x^{1/2} - 6)(2x^2 + 6)$$

$$f'(x) = \frac{4}{3}x^{5/2} + 12x^{1/2} - 4x^{-1/2} + 8x^{5/2} + 24x^{1/2} - 12x^2 - 36$$

$$f'(x) = \frac{28}{3}x^{5/2} - 12x^2 + 36x^{1/2} - 4x^{-1/2} - 36$$

### Quotient Rule:

The derivative of  $\frac{f(x)}{g(x)}$  is  $\frac{f'(x)g(x) - f(x)g'(x)}{(g(x))^2}$

1.  $f(x) = \frac{x^2 + 3}{x^2 - 4}$

$$f'(x) = \frac{(x^2 - 4)(2x) - (x^2 + 3)(2x)}{(x^2 - 4)^2} = \frac{2x^3 - 8x - 2x^3 - 6x}{(x^2 - 4)^2}$$

$$f'(x) = \frac{-14x}{(x^2 - 4)^2}$$

2.  $f(x) = \frac{x+1}{x^2 + 2x + 2}$

$$f'(x) = \frac{(x^2 + 2x + 2)(1) - (x+1)(2x+2)}{(x^2 + 2x + 2)^2} = \frac{(2x^2 + 4x + 2) - (x^2 + 2x + 2 - 2x^2 - 4x - 2)}{(x^2 + 2x + 2)^2}$$

$$f'(x) = \frac{-x^2 - 2x}{(x^2 + 2x + 2)^2}$$

3.  $f(x) = \left(\frac{x+4}{x+3}\right)(2x+5) \rightarrow$  rewrite  $\frac{(x+4)(2x+5)}{x+3} = \frac{2x^2 + 13x + 20}{x+3}$

$$f'(x) = \frac{(x+3)(4x+13) - (2x^2 + 13x + 20)(1)}{(x+3)^2} = \frac{4x^2 + 25x + 39 - 2x^2 - 13x - 20}{(x+3)^2}$$

$$f'(x) = \frac{2x^2 + 12x + 19}{(x+3)^2}$$

4.  $f(x) = \frac{x^2 + c^2}{x^2 - c^2}$

\*  $c$  is a constant

$$f'(x) = \frac{(x^2 - c^2)(2x) - (x^2 + c^2)(2x)}{(x^2 - c^2)^2} = \frac{2x^3 - 2xc^2 - 2x^3 - 2xc^2}{(x^2 - c^2)^2}$$

$$f'(x) = \frac{-4xc^2}{(x^2 - c^2)^2}$$

## Chain Rule

The derivative of  $f(g(x))$  is  $f'(g(x)) * g'(x)$

1.  $y = \sqrt{x^3 + 7}$

2.  $y = \frac{2}{(5x+1)^2}$

3.  $y = (5x^2 - 3x)^8$

4.  $y = (x^2 + 5)^3 (x^3 - 1)^4$

5.  $f(x) = \sqrt{5x^9}$

6.  $y = (9 - 3x)^{-4}$

7.  $f(x) = \frac{5}{(x^2 + 3x + 5)^3}$

8.  $y = \frac{1}{\sqrt[3]{(2x+4)^5}}$

### Chain Rule

The derivative of  $f(g(x))$  is  $f'(g(x)) \cdot g'(x)$

$$1. y = \sqrt{x^3 + 7} \rightarrow y = (x^3 + 7)^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2} (x^3 + 7)^{-1/2} \cdot 3x^2$$

$$\frac{dy}{dx} = \frac{3x^2}{2\sqrt{x^3 + 7}}$$

$$2. y = \frac{2}{(5x+1)^2} \rightarrow 2(5x+1)^{-2}$$

$$\frac{dy}{dx} = -4(5x+1)^{-3} (5)$$

$$\frac{dy}{dx} = \frac{-20}{(5x+1)^3}$$

$$3. y = (5x^2 - 3x)^8$$

$$\frac{dy}{dx} = 8(5x^2 - 3x)^7 (10x - 3)$$

$$\frac{dy}{dx} = 8(10x - 3)(5x^2 - 3x)^7$$

$$4. y = (x^2 + 5)^3 (x^3 - 1)^4 \quad * \text{ Product + Chain rule}$$

$$\frac{dy}{dx} = (x^2 + 5)^3 \cdot 4(x^3 - 1)^3 (3x^2) + (x^3 - 1)^4 \cdot 3(x^2 + 5)^2 (2x)$$

$$\frac{dy}{dx} = 12x^2(x^2 + 5)^3 (x^3 - 1)^3 + 6x(x^3 - 1)^4 (x^2 + 5)^2$$

$$5. f(x) = \sqrt{5x^9} \rightarrow (5x^9)^{1/2}$$

$$f'(x) = \frac{1}{2} (5x^9)^{-1/2} (45x^8)$$

$$f'(x) = \frac{45x^8}{2\sqrt{5x^9}}$$

$$6. y = (9 - 3x)^{-4}$$

$$\frac{dy}{dx} = -4(9 - 3x)^{-5} (-3)$$

$$\frac{dy}{dx} = \frac{12}{(9 - 3x)^5}$$

$$7. f(x) = \frac{5}{(x^2 + 3x + 5)^3} \rightarrow 5(x^2 + 3x + 5)^{-3}$$

$$f'(x) = -15(x^2 + 3x + 5)^{-4} (2x + 3)$$

$$f'(x) = \frac{-15(2x + 3)}{(x^2 + 3x + 5)^4}$$

$$8. y = \frac{1}{\sqrt[3]{(2x+4)^5}} \rightarrow (2x+4)^{-5/3}$$

$$\frac{dy}{dx} = \frac{5}{3} (2x+4)^{-2/3} (2)$$

$$\frac{dy}{dx} = \frac{10}{3} (2x+4)^{-2/3}$$

## Using Derivatives

**Sum and Difference with Derivatives:**

If  $h(x) = f(x) \pm g(x)$  then  $h'(x) = f'(x) \pm g'(x)$

Find  $\frac{dy}{dx}$ :

1.  $y = 5x^3 - 2x^2 + 3x + 5$

2.  $y = 9\sqrt{x} - \frac{1}{x^2}$

**Slope of a tangent line to a graph:**

The slope of the tangent line to a function at  $x = a$  is found by evaluating  $f'(a)$ .

Find the slope of the tangent line for:

1.  $y = 2x^2 - 3x$  at  $x = 2$

2.  $y = \sqrt{x-1}$  at  $(5, 2)$



**Equation of the tangent line to a graph at a point:**

At the point  $(a, b)$  the equation of the tangent line (in point-slope form) is  $y - b = f'(a)(x - a)$

Write an equation for the tangent line to the function at the given point in point-slope form:

1.  $f(x) = x^2 + 5x - 3$   $(-1, -7)$

2.  $f(x) = \sqrt{x+3}$   $(6, 3)$

3.  $f(x) = 3x^2(x+2)^2$   $(1, 27)$

4.  $f(x) = \frac{1}{\sqrt{x}}$   $\left(4, \frac{1}{2}\right)$

## Using Derivatives

**Sum and Difference with Derivatives:**If  $h(x) = f(x) \pm g(x)$  then  $h'(x) = f'(x) \pm g'(x)$ Find  $\frac{dy}{dx}$ :

1.  $y = 5x^3 - 2x^2 + 3x + 5$

$$\frac{dy}{dx} = 15x^2 - 4x + 3$$

2.  $y = 9\sqrt{x} - \frac{1}{x^2} \rightarrow$  *rewrite*  $y = 9x^{\frac{1}{2}} - x^{-2}$

$$\frac{dy}{dx} = \frac{9}{2}x^{-\frac{1}{2}} + 2x^{-3}$$

$$\text{or } \frac{dy}{dx} = \frac{9}{2\sqrt{x}} + \frac{2}{x^3}$$

**Slope of a tangent line to a graph:**The slope of the tangent line to a function at  $x = a$  is found by evaluating  $f'(a)$ .

Find the slope of the tangent line for:

1.  $y = 2x^2 - 3x$  at  $x = 2$

$$\frac{dy}{dx} = 4x - 3 \text{ at } x = 2$$

$$4(2) - 3 = 5$$

Slope of tangent line at  $x = 2$   
is  $\boxed{5}$ 

2.  $y = \sqrt{x-1}$  at  $(5, 2)$

$$y = (x-1)^{\frac{1}{2}}$$

$$\frac{dy}{dx} = \frac{1}{2}(x-1)^{-\frac{1}{2}} \text{ or } \frac{1}{2\sqrt{x-1}} \text{ at } x = 5$$

$$\frac{1}{2\sqrt{5-1}} = \frac{1}{2\sqrt{4}} = \frac{1}{4}$$

Slope of tangent line at  $x = 5$   
is  $\boxed{\frac{1}{4}}$

### Equation of the tangent line to a graph at a point:

At the point  $(a, b)$  the equation of the tangent line (in point-slope form) is  $y - b = f'(a)(x - a)$

Write an equation for the tangent line to the function at the given point in point-slope form:

1.  $f(x) = x^2 + 5x - 3$   $(-1, -7)$

$$f'(x) = 2x + 5$$

$$f'(-1) = 2(-1) + 5 = 3$$

↳ slope  
of tangent line

Tangent line:

$$y + 7 = 3(x + 1)$$

2.  $f(x) = \sqrt{x+3}$   $(6, 3)$

$$f(x) = (x+3)^{1/2}$$
$$f'(x) = \frac{1}{2}(x+3)^{-1/2} = \frac{1}{2\sqrt{x+3}}$$

$$f'(6) = \frac{1}{2\sqrt{6+3}} = \frac{1}{6} \rightarrow \text{slope of tangent line}$$

Tangent line:

$$y - 3 = \frac{1}{6}(x - 6)$$

3.  $f(x) = 3x^2(x+2)^2$   $(1, 27)$

$$f'(x) = 3x^2 \cdot 2(x+2)'(1) + (x+2)^2 \cdot 6x \quad (\text{product rule})$$

$$f'(1) = 3(1)^2 \cdot 2(1+2)(1) + (1+2)^2 \cdot 6(1)$$

$$f'(1) = 72 \rightarrow \text{slope of tangent line}$$

Tangent line:  
 $y - 27 = 72(x - 1)$

4.  $f(x) = \frac{1}{\sqrt{x}}$   $(4, \frac{1}{2})$

$$f(x) = x^{-1/2}$$

$$f'(x) = -\frac{1}{2}x^{-3/2} = \frac{-1}{2\sqrt{x^3}}$$

$$f'(4) = \frac{-1}{2\sqrt{4^3}} = \frac{-1}{16} \rightarrow \text{slope of tangent line}$$

Tangent line  
 $y - \frac{1}{2} = \frac{-1}{16}(x - 4)$